Geometric Electro-Magnetic Field Equations for Particles with Charge and Spin

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The complete and self-consistent "Maxwell" equations are displayed algebraically, describing (time-delayed) force and torque interactions between moving particles with Charge and Spin.

The Point

The algebra is the **4-level Grassmann linear algebra** describing 3D Euclidean space, with 4 Basis Elements **{ 3 Vectors, 1 Point }**, generating structures for bound particles, in addition to lines, planes, and volumes. Here, this is denoted as G3p1 .

With this full algebra, **Forces** from 2 or more particles can add to become a **Torque**, and two or more linear **Flows** can add to become a **Circulation** . This can not be expressed in the "tangent space" of Clifford Algebra Cl3, nor in standard Vector Algebra.

With similar clarity, the G3p1 algebra describes the motional "transformations" between vector electric $\vec{E}$ and bi-vector magnetic $\widehat{B}$, through 1st order ($\vec{v}$ / c) motion of an Origin Point, *without* 2nd order space-time algebra effects.

Thus, the algebra explicitly distinguishes between conduction currents and the (orthogonal) spin/circulation currents; and between dynamic effects such as "spin-transfer" torque, and entropic effects such as conduction resistance or magnetization damping.

Duality : It Takes Two to Tango

The triumph of Maxwell's Equations is the description of Electro-Magnetic waves, propagating outward from particles at fundamental speed **c** . These are generally viewed as either "light waves" or particle-like "Photons", expressing the "wave / particle" duality of modern physics.

Geometrically, there are two fundamental lengths in electro-magnetism :
(1) the "classical radius of the electron" $R_e = e^2/m_ec^2 = 2.82$ pm (10^{-15} m), which scales the electric interaction field E between *two* or more electric charges; and
(2) the "Compton wavelength", here denoted $D_v = \hbar c / m_ec^2 = 386.$ pm, which scales the magnetic dipole field B from a *single* electron Spin.

In G3p1, the EM wave emanating from a single particle is seen to be nilpotent (a "Photino"), with energy densiy $E^2 + B^2 = 0$. However, the *superposition of two* counter-propagating Photinos is seen to give the inter-particle Force, Torque and Helicity transfers generally attributed to a single Photon.

Understanding the complete geometry of points, vectors, bi-vectors, and tri-vectors can substantially clarify the meaning of motional transforms in space and time, and of complex probability waves.

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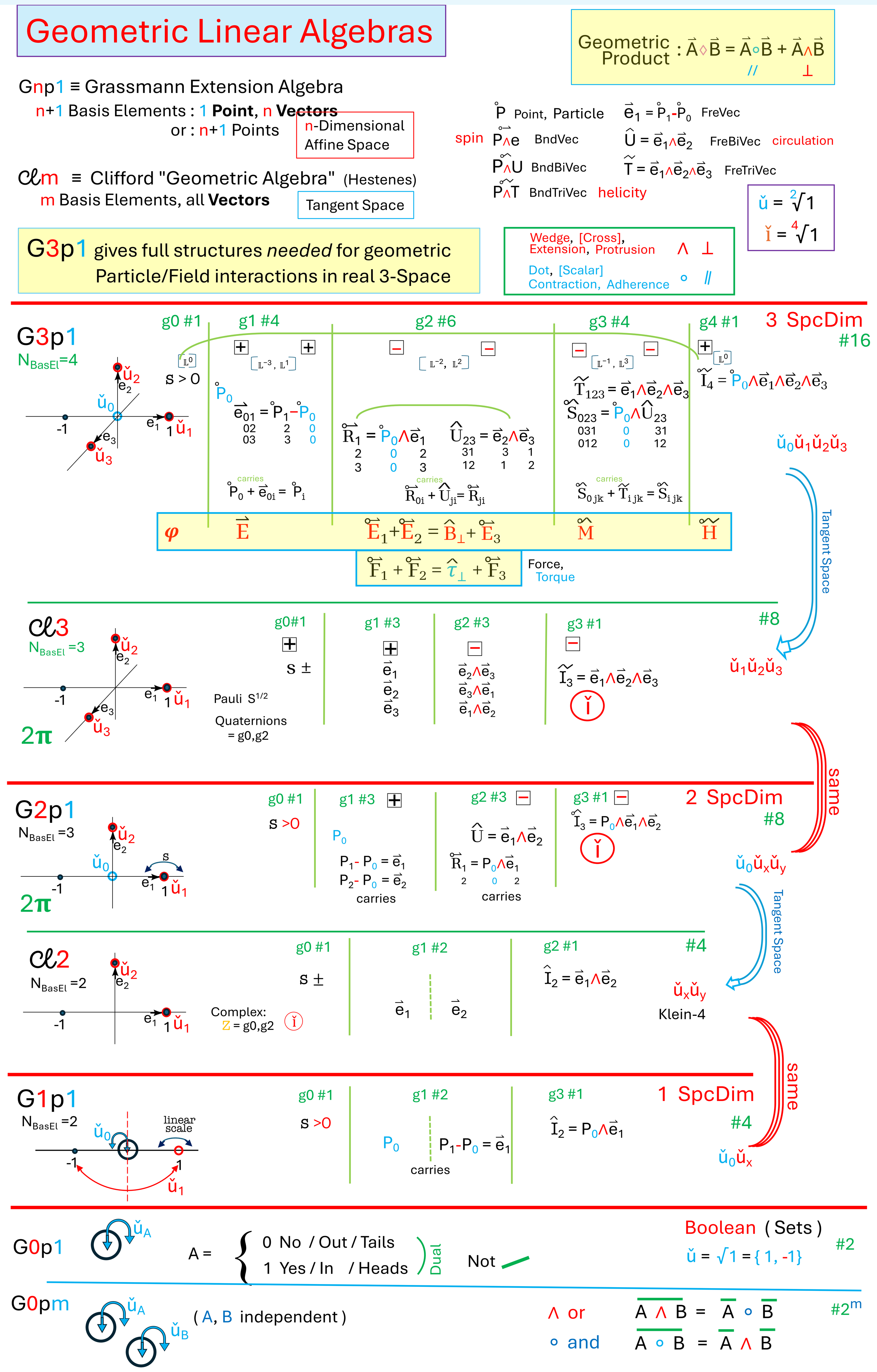
Geometric Linear Algebras



G3p1 linear algebra connects Fields to Particle Scources with Charge and Spin

	Cℓ3 Tangent	Fluid Maxwell (2-fluids)
g0	$\nabla\circ\vec{E} =$	$\nabla\circ\vec{E} = 4\pi\,(\rho_+ - \rho_-)$
g1	$-\nabla\circ\widehat{B} =$	$\nabla\times\vec{B} = \frac{\partial}{\partial ct}\vec{E} + 4\pi\,\vec{J}_{/c}$
g2	$\nabla\wedge\vec{E} =$	$\nabla\times\vec{E} + \frac{\partial}{\partial ct}\vec{B} = 0$
g3	$\nabla\wedge\widehat{B} =$	$\nabla\circ\vec{B} = 0$

$e\boldsymbol{\varphi} \sim e^2\mathbb{L}^{-1} \sim \mathcal{E}$nergy

$\vec{E} \sim e\mathbb{L}^{-2}$

$\widehat{B} \sim e\mathbb{L}^{-2} \sim e\mathbb{L}^{-1}\mathbb{T}^{-1}$

$\mathbb{L}$, $\mathbb{T}$ ⟩ c defines Time

$\mathbb{M}$ $\mathcal{E}_0 \equiv m_e c^2$

Length scales of E & M

$R_e = \dfrac{e^2}{\mathcal{E}_0}$ ~2.82 10^{-15} m

$D_v = \dfrac{\hbar c}{\mathcal{E}_0}$ ~386. 10^{-15} m

G3p1

$\check{u} = \{1, -1\}$ $\check{u}_0\ \check{u}_x\ \check{u}_y\ \check{u}_z$

$q = \check{u}_Q e$

$\vec{r}_{sf} = \mathring{P}_s - \mathring{P}_f$

$R = |\vec{r}_{sf}|$

	e−	p+	e+	p−
$\check{u}_Q$	−	+	+	−
$\check{u}_H$	+	+	−	−

field @ (x_f, t)   source @ (x_s, τ)

g0 [E] $e\boldsymbol{\varphi} = e\int dV \sum\left\{ \dfrac{q_s\mathring{P}_s}{R} + \dfrac{e\mathring{P}_s}{R}\dfrac{p_{s\parallel}}{R} \right\}_{\tau = t - R/c}$

g1 $[e\mathbb{L}^{-3}]$ $\nabla\circ\vec{E} = 4\pi \sum\left\{ q_s\mathring{P}_s + e\mathring{P}_s\dfrac{p_{s\parallel}}{R} \right\}_{\tau = t - R/c}$

g1 $\nabla\circ\widehat{B} + \frac{\partial}{\partial ct}\vec{E} = 0$

g2 $\nabla\wedge\vec{E} + \frac{\partial}{\partial ct}\widehat{B} = 0$

Waves : Oscillate Circulate

no $\vec{J}$

g3 $\nabla\wedge\widehat{B} = 4\pi \sum\left\{ \check{u}_s\mathring{P}_s \wedge \dfrac{\widehat{M}_{s\parallel}}{R} \right\}_{\tau = t - R/c}$

[E] g4 $e\widetilde{\mathcal{H}} = e\int dV \sum\left\{ \check{u}_s\mathring{P}_s \wedge \dfrac{\widehat{M}_{s\parallel}}{R} \right\}_{\tau = t - R/c}$

1D-Dipole Charge

$\vec{p}_s = (\mathring{P}_{s+} - \mathring{P}_{s-})$ $[\mathbb{L}^1]$

$p_{s\parallel} = \vec{p}_s \circ \vec{r}_{sf}$

2D-Dipole Magnetic Spin

$\widehat{M}_s = \frac{1}{2}(D_v)\, c\, \widehat{S}$ $[\mathbb{L}^2\mathbb{T}^{-1}]$

$\widehat{M}_{s\parallel} = \widehat{M}_s \circ \vec{r}_{sf}$

with $\widehat{S}_s = \check{u}_H (\vec{\beta}_s \circ \widetilde{T})$

i.e. $\widehat{S}_s \perp \vec{\beta}_s$

Positional Energy $\mathcal{E}_{/\mathcal{E}_0} = q_f\left(\boldsymbol{\varphi}(x_f) - \widehat{M}_f \circ \widehat{B} + \vec{d}_f \circ \vec{E} \right)$ (Spin 2D-Dipole Orientation; Charge 1D-Dipole Orientation)

Particle Force $\vec{F}_{f/\mathcal{E}_0} = q_f\left(\vec{E} - \widehat{M}_f \circ \nabla\widehat{B} + \vec{d}_f \circ \nabla\vec{E} \right)$

Particle Torque $\widehat{\boldsymbol{\tau}}_{f/\mathcal{E}_0} = q_f\left(- \widehat{M}_f \wedge \widehat{B} + \vec{d}_f \wedge \vec{E} \right)$

EM Waves :

$\omega / \vec{k} = c$

$\tilde{f} = \sin(\vec{k}\circ\vec{r} - \omega t)[\ \vec{E} + \widehat{B}\ ]$

$\vec{k} \perp \vec{E}$, $\widehat{B} \parallel \vec{k}$

Invariants :

$\mathcal{E} = \vec{E}^2 + \widehat{B}^2$ Energy

$\widetilde{\mathcal{H}} = \vec{E} \wedge \widehat{B}$ Helicity

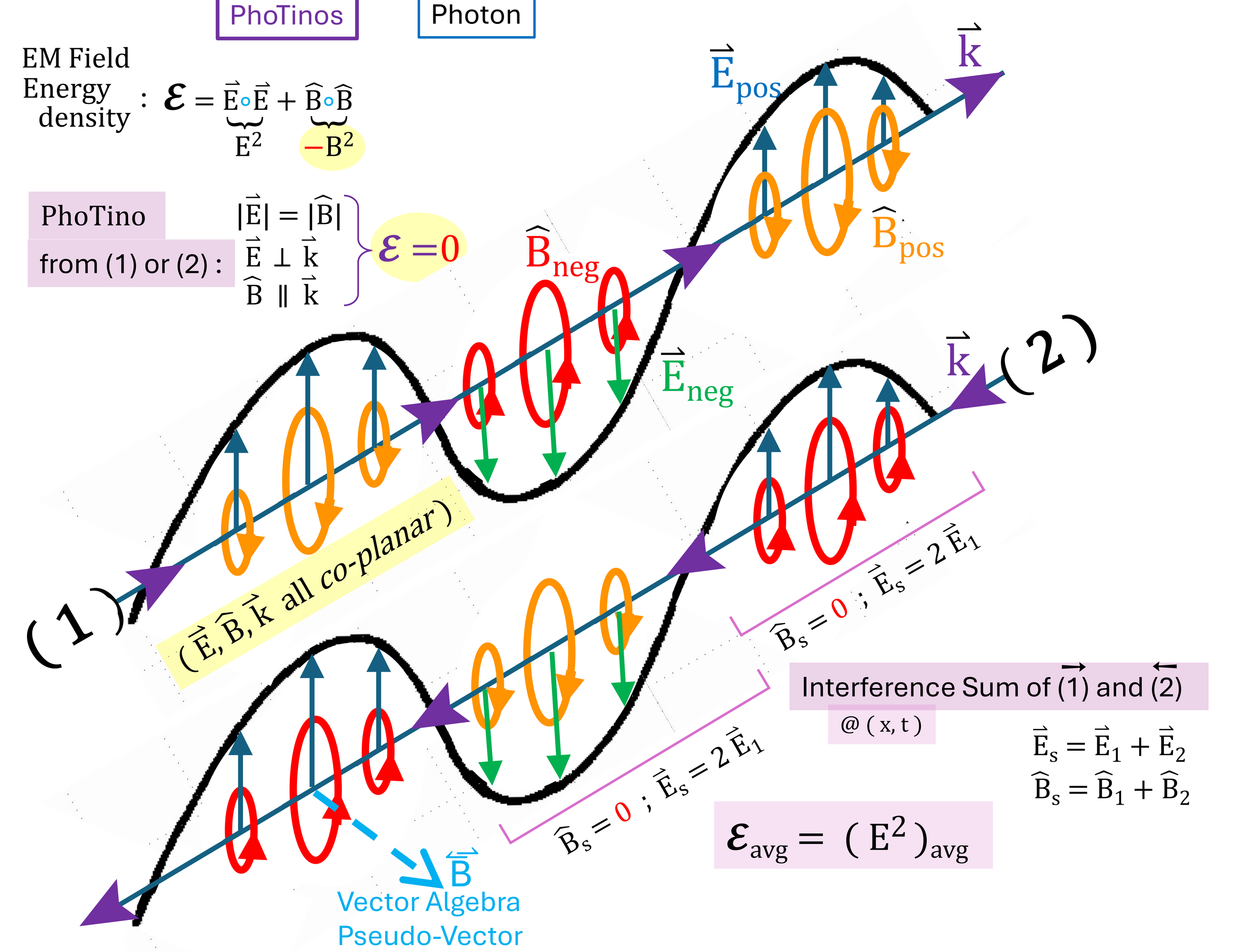
Nilpotent "Photino"

$\widehat{B} = \check{u}_H \vec{k} \wedge \vec{E}$ $\mathcal{E} = 0$

co-planar : $\widehat{B} \parallel \vec{E}$

It Takes Two to Tangle

PhoTinos   Photon



Particles (1),(2) emanate "NilPotent" spherical waves w1,w2 ; waves w1,w2 give simultaneous Forces on particles (1),(2) .